Brass Tacks

An in-depth look at a radio-related topic

Impedance

When we venture into building amateur radio gear, such as an antenna, balun, or even a tuner, we'll likely encounter the notion of *impedance*. Even if we're not into making stuff, we might still run into this word, since it seems to be coupled with how we connect our equipment. But *just what is it*, and more importantly, *why do we need to know about it?* In a separate article, we touched briefly on impedance as an effect inherent to passive components, but this discussion focuses on the property itself.

In the amateur radio world, electrical impedance is the property of opposition to (reduction of) electrical current flow, expressed in *ohms*, symbol Ω. It's represented by the mathematical expression

$$Z = R + jX$$

in which **Z** represents impedance, **R** is resistance, **j** is the imaginary unit (square root of –1), and **X** represents reactance.

Resistance is simply the opposition to current flow, without regard to frequency, and it's fairly easy to grasp because it's part of the familiar Ohm's Law

$$E = I \times R \text{ or } I = E \div R$$

In other words, the current through a resistive component is related to the electrical pressure (voltage) placed on it, and the component's ability to resist the pressure.

As you can see from the expression, impedance is a complex number, meaning that it has a *real* portion and an *imaginary* portion, and that imaginary part is what tends to make understanding it a little challenging. Especially when you consider that reactance, the imaginary part, is further defined by whether it arises from inductance or capacitance, and is defined as

$$X = X_L - X_C$$

in which the two reactance quantities are defined as

$$X_L = 2\pi fL \text{ and } X_C = 1/(2\pi fC)$$

in which **f** is the frequency, **L** is the inductance, and **C** is the capacitance. This makes the total impedance equation

$$Z = R + j[2\pi fL - 1/(2\pi fC)]$$

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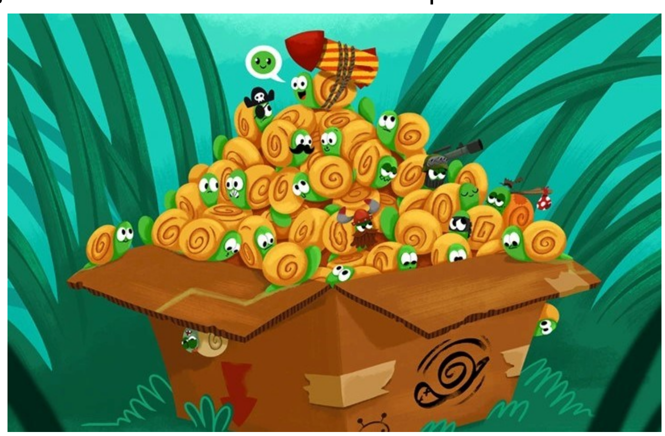
meaning that impedance can be frequency-dependent. Also, because impedance is a complex number, there exists another complex number that acts as its counterpart, known as its *complex conjugate*, such that, if **Z = R + jX**, its complex conjugate (denoted ***Z****) is

$$\mathbf{Z^* = R - jX}$$

and the only difference being that the reactances are opposite in sign. As will be explained later, this complex conjugate can prove useful in some applications.

Impeded flow

Electrical impedance is not related to the speed of anything, so electron speed or even energy speed does not affect impedance, and vice versa. The speed of electricity (electrical energy) depends solely on the material ("medium") through which it travels. So, exactly what is being impeded? It's the volume of electrical charge (number of electrons) that passes through a particular point per second, also known as the flow of electrical current, not how fast they're moving. Speed is measured in length per second, but flow is measured in volume per second.



To better visualize this, let's slow things down to a snail's pace. Say we have a box full of snails, and we want them all to go through a medium-sized hole in our fence to the field out in back, so that they can only go through about three or four at a time. No matter how big or small the hole is, the snails all travel at the same speed (except the one with the rocket), yet the size of the hole restricts the number of them that can get through at any given moment.

When I apply pressure (voltage) on the snails by tempting them with beer (they love the smell of the cooked yeast) in the field, they begin their march through the hole. And the number of them going through at any time (current) is again limited by the size of the hole (impedance). Not a perfect analogy, but I believe you get the idea. Even if I attempt to pressure the snails to go through in greater quantities, they still move at the same speed – a snail's pace.

Resonance

It's difficult to mention impedance without mentioning resonance, since resonance depends on impedance. In fact, passive electrical resonance is no more than a state in which a network of reactive components (inductors and capacitors) exhibits an impedance consisting only of the resistance of the wires; that is, with the reactances completely canceled, and the resistance very small. Because this phenomenon occurs at a unique *resonant frequency*, it's useful in amateur radio for use as a *tuned circuit*, to allow you the reception of signals from a particular band (range) of frequencies, while filtering out all others.

To demonstrate how this might be useful, let's design a *trap antenna*; that is, an antenna de-

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signed for one band, but uses a device called a *trap* to allow it to act like an antenna on another band. For convenience (and simplicity), let's design the trap for a 40-meter dipole, so that it's also resonant on 20 meters. To achieve that, let's assume the trap inhibits all signals except those for 40 meters, This means the trap acts like a wire (0-ohm short circuit) at 40 meters, say, 7.2 MHz. Performing the math for impedance, then, yields

$$Z = R + jX = 0 \text{ ohms}$$

Since the wire is short enough (about 65 feet) to assume that its resistance R is also 0 ohms,

$$0 = 0 + jX \text{ ohms}$$

therefore, **X** = **0**, but

$$X = X_L - X_C = 2\pi fL - 1/(2\pi fC) = 0$$

$$2\pi fL = 1/(2\pi fC)$$

Then, solving for **C**, we have

$$C = 1/(4\pi^2 f^2 L)$$

If we create an inductor out of a length of coiled wire, and measured it on an analyzer, let's say it displays around 40.7 µH of inductance. The capacitor value is then

$$C = 1/[4\pi^2(7.2 \text{ MHz})^2(40.7\ \mu\text{H})] = 12 \text{ pF}$$

Therefore, cut the 65-foot wire in half and place a 12 pF capacitor in series with your 40.7 µH coil, and you've got yourself a 20-meter / 40-meter trap antenna. It'll be close to the correct length for a 20-meter dipole antenna, and then perform perfectly on 40 meters when you attempt to transmit on 40 meters because of the “short circuit” you've created at that frequency.

Characteristic impedance

All electrically conductive things possess impedance, and more often than not, their impedances are defined by the types and geometries (sizes and shapes) of the materials they're made of. But their impedances also vary with frequency, making many conductors unsuitable as transmission lines, which are wires or other conductors that carry RF (radio frequency) electrical signals between the transceiver and the antenna.

We can design a transmission line such that its impedance remains fairly constant for a large workable frequency range and length, and we call this constant impedance the *characteristic impedance*. Amateur radio feed lines, such as coax (coaxial cable), window line, and ladder line are constructed in such a way, and each exhibits a characteristic impedance that's fairly constant over a large, workable frequency range.

By definition, characteristic impedance is that of the input for an infinitely long transmission line. Another way to look at it is the value of resistance that, when used as a termination

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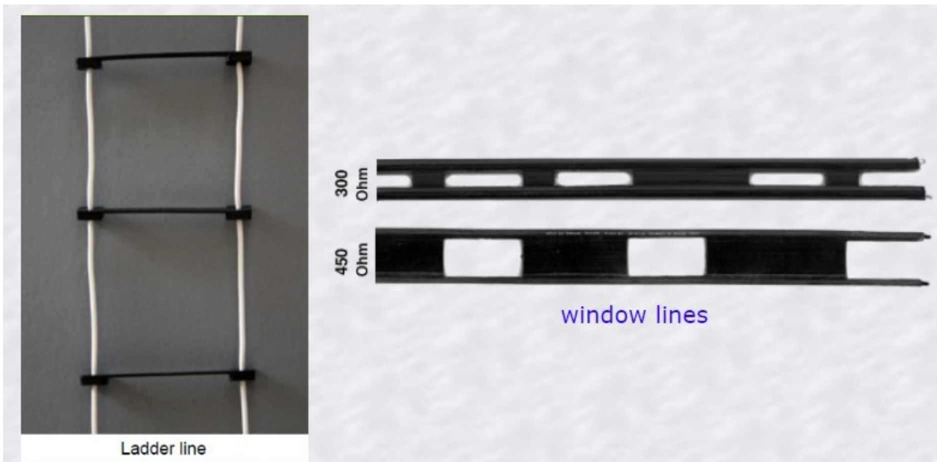
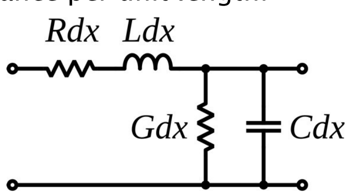


(called the load) at the output of the line, makes the input impedance of the line independent of the electrical length of the line. The typical characteristic impedances of the examples I gave are 50 ohms for coax, 300 ohms for mini-window line, 450 ohms for regular window line, and 600 ohms for open ladder line, also called open wire line.

But just how is the impedance of coax 50 ohms? The following diagram generalizes the appearance of almost any transmission line, such as coax, in which ***Ldx*** is the inductance per unit length and ***Cdx*** is the capacitance per unit length:



The cable does indeed exhibit some resistance, labeled ***Rdx***, but that value tends to be rather small, typically less than a third of an ohm for 100 feet of RG-8X and less than a tenth of an ohm for 100 feet of LMR-400, for example. The conductance, labeled ***Gdx***, is the reciprocal of the huge resistance (as will be described later, under **Admittance**) between the center conductor and shield, so tends to be very small, in the order of micro-siemens per foot. The generalized equation for transmission line impedance is defined as shown on the left:

$$Z = \sqrt{\frac{R + sL}{G + sC}} \qquad\qquad Z_0 = \sqrt{\frac{sL}{sC}} = \sqrt{\frac{L}{C}}$$

In this case, ***s*** is the imaginary unit and frequency, or j2πf. If ***R*** and ***G*** are as small (effectively zero) as we believe, then the frequencies (the two ***s*** variables) cancel, and we have $\boldsymbol{Z_0}$, the characteristic impedance as shown on the right. For example, given that RG-8X cable exhibits 0.077 µH/ft and 30.8 pF/ft, the $\boldsymbol{Z_0}$ for RG-8X = √[(0.077 µH/ft)/(30.8 pF/ft)] = 50 ohms.

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But if the frequency values cancel (because the cable resistance and conductance values are so low), it means this characteristic impedance holds, regardless of frequency. In real-life, however, an actual cable does have an upper usable frequency limit. RG-8X can by used to 1.0 GHz and LMR-400 can be used to 6.0 GHz, for example, depending on the manufacturer.

Maximum power transfer

Let's connect a transceiver to a transmission line (coax) input and "look" into the output of the line back toward the transceiver at the antenna end. The impedance we should "see" at the output end is the **lumped sum** of that of the transceiver and that of the transmission line, and let's call it Z_s, the two impedances combined as the *source impedance*. Let's then place our load impedance (Z_L, which might be an antenna) on the antenna end, the output of the line.

We now have essentially a circuit with the source voltage, the source impedance, and the load impedance. The question of interest here is, What value of Z_L can give us the best possible transfer of power from the transceiver to the load (antenna)?

If we do the math (not at this time, sorry), we find that a very high value of Z_L gives us the best efficiency; that is, the most amount of power going out the antenna for a given amount of power coming from the transceiver. But that "most amount of power" will be quite small, due to the large load impedance. On the other hand, a very low value of Z_L gives us the worst efficiency because most of the power will be dissipated in the transceiver instead of the antenna, possibly placing our transceiver finals in danger.

It turns out that the most amount of power we can get from our transceiver to our antenna, known as *maximum power transfer*, occurs when the source and load impedances match.

Notice I said "match" instead of "are equal" because "equal" only applies if the two impedances are purely resistive, which they rarely are. In fact, the word "match" means the two impedances are *complex conjugates* of each other, again an easy proof, but I'll spare you the unnecessary math. This means if the source impedance, as seen from the antenna end is **R + jX**, then the antenna must present an impedance of **R – jX** to allow for the maximum amount of power out of it from the transceiver. But we typically have the opposite problem, in that our antenna exhibits an impedance of **R + jX** ohms. How do we make our transceiver-and-feed line 50-ohm source match the antenna's **R + jX** load?

To perform the match between your transceiver and your antenna, you can create a matching network, such as a T network, pi network, or pi-L network, which all work very well, but they're only good for a small frequency range. Or you can use a *tuner*. The job of a tuner is to present a 50-ohm load to the transceiver and a **R – jX** load to your antenna. And it's adjustable, so its impedance matching isn't confined to a small range. Then again, you can use variable capacitors and variable inductors to perform the match in your home-made network, but today, a tuner is just plain convenient.

Impedance and SWR

The SWR (standing wave ratio) is the value of the maximum voltage standing wave amplitude compared with the value of the minimum amplitude. Standing waves arise due to *reflections* of a signal in a transmission line because of imperfect matching between the load (antenna) impedance and the lumped source impedance. Because SWR is calculated from the comparison of

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the incident (forward-traveling) signal and the reflected (reverse-traveling) signal, we start by a quantity known as the reflection coefficient (Γ):

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \quad \text{and} \quad SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

It follows that

$$SWR = \frac{1 + \left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|}{1 - \left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|} = \frac{\frac{Z_L + Z_0 + Z_L - Z_0}{Z_L + Z_0}}{\frac{Z_L + Z_0 - Z_L + Z_0}{Z_L + Z_0}} = \frac{2Z_L}{2Z_0} = \frac{Z_L}{Z_0}$$

It turns out, therefore, that the SWR of an antenna can be calculated using impedances alone, and is simply ***the ratio of the antenna impedance to the feed line characteristic impedance***, or the ratio of the characteristic impedance to the antenna impedance, whichever is larger (to satisfy the absolute value condition).

A practical application

Many hams have discovered the value in long wire and EFHW (end-fed half-wave) antennas, because of their effectiveness and yet simple construction. The big problem with a long piece of wire, however, is that its natural impedance tends to be much higher than most amateurs are accustomed to. Even if you attempt to cut the wire to length according to formulas, their impedances typically range from 1800 ohms to 5000 ohms for HF frequencies.

It turns out that we can construct a transformer that converts the impedance of such a wire by matching its impedance on one side of the transformer and presenting a more usable impedance on the other. This technique of impedance matching is similar to what your tuner does, but to match this huge impedance difference is outside the ability of most tuners. The goal, then, is to design a transformer with a primary impedance of 50 ohms, and a secondary impedance of between 1800 ohms and 5000 ohms.

The transformer calculation will involve determining the number of required winding turns on each side, to match the impedances on both sides, since the voltages and currents are not only unknown, but unnecessary. We start by relating the numbers of turns **N** to impedances **Z**:

$$\frac{Z_S}{Z_P} = \frac{\frac{V_S}{I_S}}{\frac{V_P}{I_P}} = \frac{V_S \times I_P}{V_P \times I_S} \qquad V_S = V_P \times \frac{N_S}{N_P} \text{ and } I_P = I_S \times \frac{N_S}{N_P}$$

This is from the transformer relationship between turns ratios and voltages and currents. Continue substituting and canceling results in

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$$\frac{Z_S}{Z_P} = \frac{\left(V_P \times \frac{N_S}{N_P}\right) \times \left(I_S \times \frac{N_S}{N_P}\right)}{V_P \times I_S} = \frac{N_S^2}{N_P^2} = \left(\frac{N_S}{N_P}\right)^2$$

This means the impedances relate to each other by the square of the turns ratio. In the case of the long wire, the squares of the turns ratios are

$$\frac{5000\text{ ohms}}{50\text{ ohms}} = 100 \text{ and } \frac{1800\text{ ohms}}{50\text{ ohms}} = 36$$

For convenience, let's select a perfect square between those values, such as 64 or 49. Selecting 49, for example, will result in standing wave ratios of

$$\frac{1800\text{ ohms}}{49} = 37\text{ ohms, and } \frac{5000\text{ ohms}}{49} = 102\text{ ohms}$$

which in turn result in

$$\frac{50\text{ ohms}}{37\text{ ohms}} = 1.35:1\text{ SWR, and } \frac{102\text{ ohms}}{50\text{ ohms}} = 2.04:1\text{ SWR}$$

both well within most internal tuner (maximum 3.0:1 SWR) range. The required number of transformer windings are

$$\sqrt{\frac{Z_S}{Z_P}} = \sqrt{49:1} = 7:1$$

and then wind it with 14 turns on one side and 2 turns on the other (same 7:1 ratio) to improve mutual inductance while keeping ohmic losses (resistances) low.

Admittance

In a number of cases (as in the *characteristic impedance* example above), it's convenient to refer to the reciprocal of impedance, known as ***admittance***. This can be expressed as

Y = 1/Z

Just as the formula for impedance is

Z = R + jX, the formula for admittance is **Y = G + jB**

in which **Y** is the admittance, **G** is known as conductance, **B** is known as susceptance, and **j** is

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the same imaginary unit. Therefore, in terms of impedance, admittance is also

$$Y = 1/(R + jX)$$

and is the property of allowance (opposite of opposition) for electrical current flow, expressed in siemens, symbol S. At one time, it was proposed that the unit for admittance be the mho, which is backwards for ohm, but that idea never caught hold for long in either academic, professional, or SI circles, and was eventually dropped.

Polar form

From our discussion above, impedance is defined as the complex value

$$Z = R + j[2\pi fL - 1/(2\pi fC)]$$

which is its Cartesian ("rectangular") form. But, it can also be expressed in polar form, which is convenient for some calculations. Essentially, it is

$$Z = Z_{mag}\underline{/\varphi}$$

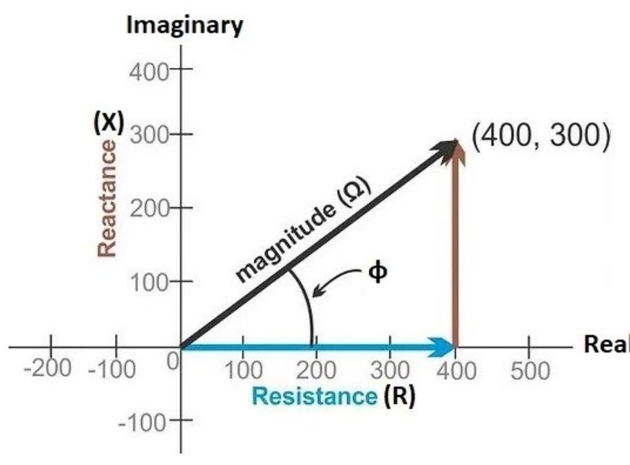
in which $Z_{mag} = \sqrt{(R^2 + X^2)}$ and $\varphi = \tan^{-1}(X/R)$, in which Z_{mag} is known as the *magnitude* and φ is the *phase angle*. To convert back to Cartesian format,

$$Z = Z_{mag}\cos\varphi + jZ_{mag}\sin\varphi$$

And converting between impedance and admittance is simply a matter of taking the reciprocal of Z_{mag} and reversing the sign of the angle. Therefore and impedance of $4\underline{/30^\circ}$ ohms equates to an admittance of $0.25\underline{/-30^\circ}$ siemens.

The following graph represents a circuit impedance (400 + j300 ohms) in rectangular coordinates, for which the horizontal axis represents **R**, the resistance or the "real" component, and the vertical axis represents **X**, the reactance or "imaginary" component. This impedance is represented by using a positive **R** value (all resistances are positive values) and a positive **X** value.

But the graph also shows its polar equivalent if we draw a diagonal line from the origin to the end point of the **X** value, creating an angle **φ** between the **R** value and the diagonal line. Therefore, the impedance in this case is **Z = R + jX**, and is the same impedance is represented as $Z_{mag}\underline{/\varphi}$. Just as **X** being positive tells you that the represented impedance is *inductively reactive*, the positive phase angle tells you the same. Had this reactance been negative, the angle would also have been negative, and the **X** segment would be drawn downward, below the horizontal axis, indi-



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cating that the impedance is *capacitively reactive*.

Finally

Let's get over a few pedantic semantics:

- Resistance is impedance. Reactance is impedance. The complex combination of resistance and reactance is impedance.
- Impedance is the opposition to current flow, whether that flow is in the form of alternating current, direct current, or some complicated mixture of signal types, such as digital, pulse, or a transient.
- The imaginary unit ***j*** is used here and in many texts instead of ***i*** to avoid confusion with the conventional ***i(t)***, which represents a time-varying current.

Also, we've omitted several relevant topics, such as complex power, AC circuit analysis, and parallel circuits, whose impedance calculations are important, but have limited this discussion in the interest of brevity. In fact, while writing this article, it became clear to me that impedance is such a large subject that I had to go back and remove some of the content, including some good examples. So, if you're wondering why I haven't mentioned your favorite impedance issue, it might just be my attempt at keeping this already-lengthy discussion to a sane length.

Summary

Electrical impedance is the opposition to current flow, and its value is represented by a complex number, combining resistance and reactance. Impedance affects the amount (volume) of electrons that's allowed to go through a component per second, and not their speed. It's the natural opposition to current flow of a conductor that's inherent to its material. Characteristic impedance is that due to the circuital makeup of a combination of conductors, such as coax. Non-unity SWR is due to the potential signal reflections from a load whose impedance does not match that of its transmission line, and so can be calculated from a ratio between the two. An impedance-matching network or device such as a transformer or a tuner can be inserted between the transmitter and the antenna to bring the antenna system to resonance and allow for maximum power transfer. Admittance is the reciprocal of impedance, simplifying some calculations. Both impedance and admittance can be expressed in polar, as well as Cartesian (rectangular), form.

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